**Syllabus**

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| Discipline’s code | Discipline’s title |  | | | Number of ECTS | SWST  Self-work of student with teacher in hours | SWST  Self-work of student without teacher in hours |
| Lect. | Pract. | Lab. |
|  | **Modelling of Environmental Processes and Management** | 64 | 65 | | **12 ECTS (360 hours)** | 22 | 209 |

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| Academic presentation of the course | **Aim of the course:**  The aim of the course is to introduce mathematical modelling as a process analysis tool. Using mathematical modelling principles, a mechanism for the study and analysis of classical dynamical systems is presented. Numerical methods for solving such problems are explored. This establishes a universal approach to various dynamical systems: the appropriate mathematical model is discussed, quantitative and qualitative analysis is carried out, and the obtained results are reviewed and analysed. Mathematical models of some environmental processes are used for illustrations.  **As a result of studying the discipline, students should be able to:**   1. Understand of how to create and apply mathematical models. 2. Acquire general knowledge of dynamical systems. 3. Choose the appropriate numerical method for the solution of a particular problem. 4. Analyze of the obtained results and  evaluate suitability. |
| Prerequisites | Differential and integral calculus basics, introduction to numerical methods |
| Post requisites |  |
| Information resources | **Literature**:   1. Smith, Jo U. Introduction to environmental modelling. Oxford: Oxford University Press, 2007. 2. M.K Theodore, L. Theodore Introduction to Environmental management. CRC Press, 2009. 3. Lennart E. Introduction to computation and modelling for differential equations. Hoboken, NJ: John Wiley&Sons, 2016. 4. Edwards, C. H.; Penney, D. E. Differential Equations. Computing and Modeling. 4th edition. Upper Saddle River: Pearson Prentice Hall, 2008. 5. S.H. Strogatz. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.Westview Press, 2014. 6. M.W. Hirsch, S. Smale, R.L. Devaney. Differential equations, dynamical systems and an introduction to chaos. Academic Press, 2013. 7. Kreyszig E. Advanced engineering mathematics. Hoboken, NJ: John Wiley&Sons, 2011 8. Zill, D. A First Course in Differential Equations with Modeling Applications. Belmont: Brooks/Cole, 2012. 9. Numerical Methods for Engineers. By Steven Chapra and Raymond Canale. 7th edition. McGraw-Hill Education, 2014.   **Internet-resources:**  1. Mervin D. Palmer. Water quality modelling and prediction. <https://ecommons.cornell.edu/bitstream/handle/1813/2804/>  [12\_chapter12.pdf;jsessionid=5B205919799F4BE50884C26FAEA547E9?sequence=9](https://ecommons.cornell.edu/bitstream/handle/1813/2804/)  2. S.P. Kuznetsov. Dynamical chaos. Fizmatlit, Moscow, 2001 (in Russian). <http://www.ereading.club/book.php?book=107806>  3. Holistic Numerical Methods. <http://numericalmethods.eng.usf.edu>  4. Applied Engineering Analysis: by Tai-Ran Hsu. Published by John Wiley & Sons, 2018. <https://www.sjsu.edu/me/docs/hsu-Chapter%2010%20Numerical%20solution%20methods.pdf> |

Calendar (schedule) the implementation of the course content**:**

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| --- | --- | --- | --- |
| Week / date | Topic title (lectures, practical classes, Independent work of students, IWS) | Number of hours | Maximum score |
| 1 | 2 |  | 4 |
| 1 | Lecture  General principles of mathematical modelling. | 2 | 1 |
| Lecture  Mathematical modeling application examples | 2 | 1 |
| Practical class (with computer)  Analysis of prepared mathematical models using computer programs | 2 | 1 |
| 2 | Lecture  Equations of motion: creation and analysis | 2 | 1 |
| Lecture  Equations of fluid dynamics. Application of Bernoulli and Torricelli's laws in fluid dynamics | 2 | 1 |
| Practical class  Creation and analysis of differential equations describing simple movements of bodies. | 3 | 1 |
| Practical class (with computer)  Describing of bodies movement by equations and solving them using computer software. Application of Bernoulli and Torricelli's laws | 2 | 1 |
| 3 | Lecture  Creation and analysis of population dynamics models | 2 | 1 |
| Lecture  Maltus and Verhulst equations. Logistics models | 2 | 1 |
| Practical class  Application of various models of population dynamics and analysis of the obtained results | 2 | 1 |
| Practical class (with computer)  Analysis of ecological systems. Model parameter influence analysis | 2 | 2 |
| 4 | Lecture  Simple mixture modeling and radioactive decay modeling problems | 2 | 1 |
| Lecture  Models of pollution dynamics in water bodies | 2 | 1 |
| Practical class  Modeling and analysis of problems of mixtures, radioactive decay and pollution dynamics | 2 | 1 |
| Practical class (with computer)  Modeling of heat transfer in a body immersed in a liquid | 2 | 1 |
| Self-work (Self-study) of student with teacher: SWST.  Independent creation and analysis of a model (for example, modeling of a liquid flowing from a vessel of a certain shape and determining the height of the liquid in the vessel at any time) | 2 | 5 |
| 5 | Lecture  Generalization: first-order continuous dynamic systems and their research methods | 2 | 1 |
| Lecture  Types of stability points. Analysis in a straight line and a circle. Bifurcations and their types | 2 | 1 |
| Practical class  Qualitative analysis of mathematical models analyzed in Lectures 2 – 4; Ecosystem models; Laws of Cooling; equations of motion | 2 | 1 |
| Practical class (with computer)  Creation and analysis of phase portraits of first-order dynamic systems. Ecosystem models; Laws of Cooling; equations of motion | 2 | 1 |
| Self-work of student with teacher: SWST.  Independent and detailed quantitative and qualitative analysis of chosen dynamic system | 2 | 5 |
| 6 | Lecture  Processes modeled by systems of differential equations. Simplest models of communicating vessels. | 2 | 1 |
| Lecture  Models of different types of oscillations. Models of competitive species | 2 | 1 |
| Practical class  Quantitative analysis of some systems of differential equations. | 3 | 1 |
| Practical class (with computer)  Creating competitive species models and solving them using computer software | 2 | 1 |
| 7 | Lecture  Qualitative study of linear second-order dynamic systems. Stability points and their classification. Phase portraits and their analysis | 2 | 1 |
| Lecture  Qualitative analysis of nonlinear second order dynamic systems. Nonlinear system linearization, stability points | 2 | 1 |
| Practical class  Development and analysis of a second order linear dynamic system.  Research of second order nonlinear dynamic systems | 2 | 1 |
| Practical class (with computer)  Analysis of second order dynamic systems: stability points, phase portraits, dynamics analysis | 2 | 1 |
| Self-work of student with teacher: SWST.  Development and analysis of a second order dynamic system (ecological models) | 4 | 5 |
| 8 | Lecture  Nonlinear dynamic systems in a polar coordinate system. Signs of absence of closed orbits | 2 | 1 |
| Lecture  Limit cycles. Bifurcations in second order dynamical systems | 2 | 1 |
| Practical class  Application of the criteria for the presence (absence) of limit cycles. | 2 | 1 |
| Practical class (with computer)  Bifurcation study. | 2 | 1 |
| 9 | Lecture  Numerical methods and their importance. Approximate calculations. | 2 | 1 |
| Lecture  Solving of nonlinear equations by numerical methods | 2 | 1 |
| Practical class  Working with approximations | 2 | 1 |
| Practical class (with computer)  Solving nonlinear equations by numerical methods. | 2 | 1 |
| 10 | Lecture  Solving systems of nonlinear equations by numerical methods. | 2 | 1 |
| Lecture  Solving systems of linear equations by numerical methods: direct and iterative methods | 2 | 1 |
| Practical class (with computer)  Solving systems of nonlinear equations by numerical methods. | 2 | 1 |
| 11 | Lecture  Solution of tridiagonal linear systems | 2 | 1 |
| Lecture  Iterative methods for solving systems of linear equations | 2 | 1 |
| Practical class (with computer)  Solving systems of linear equations by numerical methods | 2 | 1 |
| Practical class (with computer)  Solution of tridiagonal linear systems. Applications. | 2 | 1 |
| 12 | Lecture  Numerical integration | 2 | 1 |
| Lecture  Methods of numerical integration: rectangle and trapezoidal formulas, Simpson's formulas. Numerical integration errors. Gauss formulas | 2 | 1 |
| Practical class (with computer)  Methods of numerical integration and their application | 2 | 1 |
| Self-work of student with teacher: SWST.  Calculation of one integral by several different methods, estimation of errors, analysis of results. | 2 | 1 |
| 13 | Lecture  Solving differential equations by numerical methods. Convergence of solutions, stability, accuracy of approximation. | 2 | 1 |
| Lecture  Euler methods, Runge–Kutta methods | 2 | 1 |
| Practical class (with computer)  Analysis of first-order dynamic systems using numerical methods. | 3 | 1 |
| Practical class (with computer)  Solving of first-order dynamic systems using standard solvers | 2 | 1 |
| Self-work of student with teacher: SWST.  Solving a first-order dynamic system by numerical methods manually and using computer software. | 4 | 5 |
| 14 | Lecture  Solving systems of differential equations by numerical methods: approximation. Finite differences and Runge-Kutta methods | 2 | 1 |
| Lecture  Crank - Nicolson and alternating directions methods | 2 | 1 |
| Practical class (with computer)  Solving systems of differential equations by numerical methods | 2 | 1 |
| Practical class (with computer)  Solution of higher order dynamic systems using standard solvers | 2 | 1 |
| Self-work of student with teacher: SWST.  Numerical analysis of a competitive species model. | 2 | 5 |
| 15 | Lecture  Water flow modeling: basic processes and their mathematical models. Static models, kinetic reactions. | 2 | 1 |
| Lecture  Material balance equation. Advection, diffusion equations. Reaction-diffusion equations. | 2 | 1 |
| Lecture  Two-dimensional pollutant flow model. Initial and boundary conditions. | 2 | 1 |
| Lecture  Approximation of water quality modeling problems and solving by numerical methods | 2 | 1 |
| Practical class  Analysis of the mathematical models of the various problems associated with water quality | 2 | 1 |
| Practical class (with computer)  Modeling of nitrogen, oxygen and phytoplankton cycling | 2 | 1 |
| Practical class (with computer)  Modeling of pollutant flow in a river section (1D case): different cases of boundary conditions and their influence on solutions | 2 | 1 |
| Practical class (with computer)  Modeling of pollutant flow in a river section ( 2D case) | 2 | 1 |
| Practical class (with computer)  Modeling of the dispersion of chemical pollutants in the coastal zone | 2 | 1 |
| Practical class (with computer)  Reaction-diffusion equation for radioactive materials | 2 | 1 |
| Self-work of student with teacher: SWST.  Detailed analysis of one applied problem. | 6 | 6 |
|  |  | 151 | 100 |